# X(3872): Hadronic Molecules in Effective Field Theory

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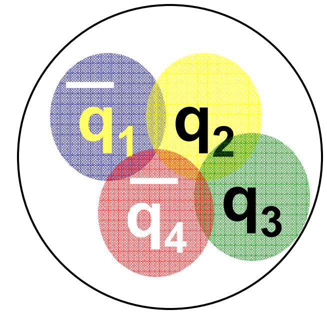
# Introduction: why do we care?




1. Multiquark states: do they exist?

   QCD: Yang-Mills theory based on SU(3) gauge group
   Quarks: fundamentals ("color triplets"): 3
   Antiquarks: antifundamentals: 3

   Mesons: $3 \times \bar{3} = 1 + 8$

   Baryons: $3 \times 3 \times 3 = (6 + \bar{3}) \times 3 = 10 + 8 + 8 + 1$

   Others? In principle, yes...

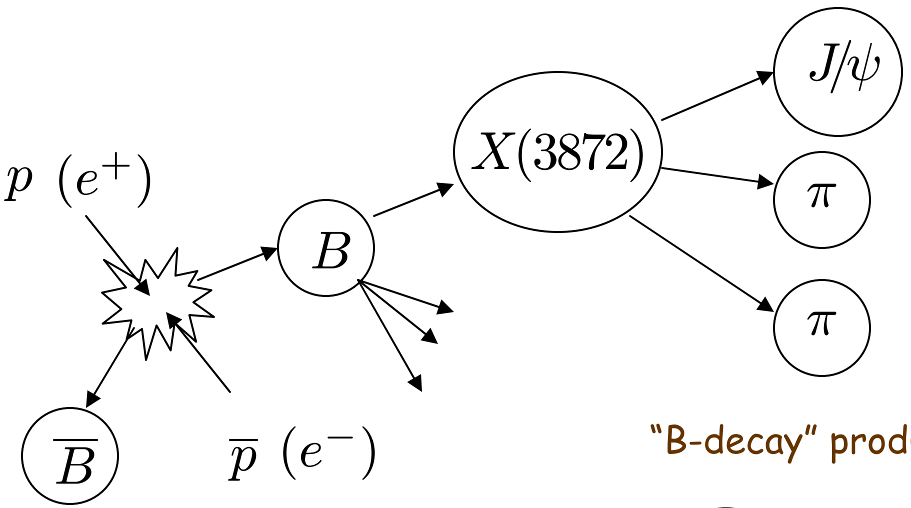
2. In practice: do not know !!! The answer implies understanding of non-perturbative effects in QCD!

3. N. Isgur: quark models plus coupled final state channels dynamically DISFAVOR multiquark states...

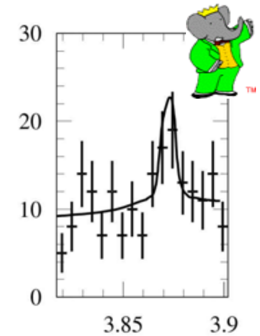
   2 or 3 quark states or "fall-apart" states are energetically more favorable

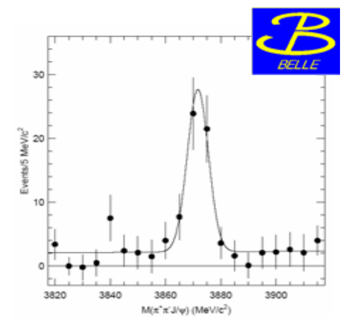
# X(3872) at e⁺e⁻ and hadron colliders

X(3872) was first observed by Belle collaboration

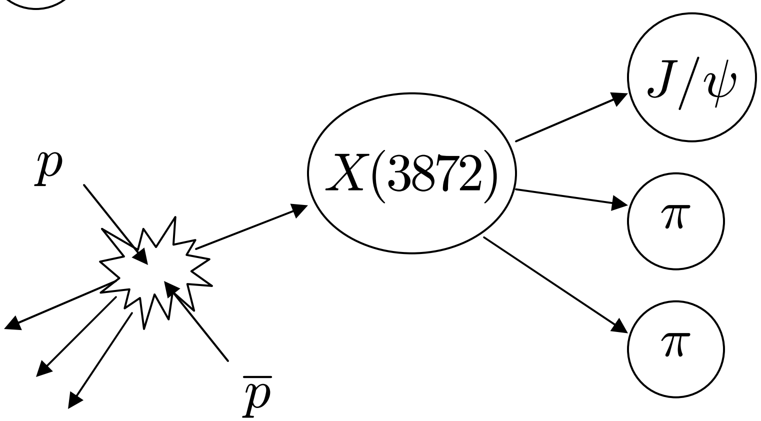


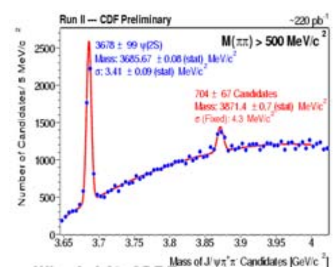

"B-decay" production

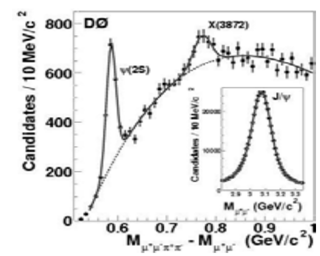



... and confirmed by BaBar...




Prompt production

... and then CDF and D0

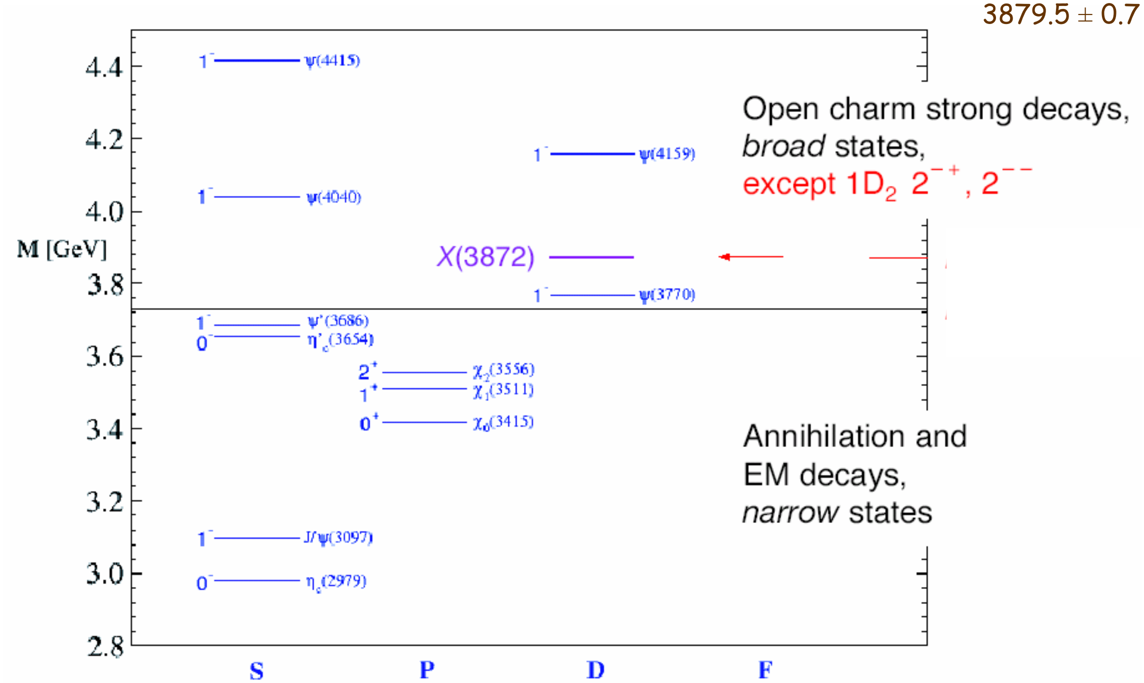
**CDF: only ~16% of X(3872) are produced in B-decays**

# What is so special about X(3872)?

X(3872) possesses several curious features:

$m(D^0+\bar{D}^{0*})=$
3871.5 ± 0.5 MeV
$m(D^++D^{-*})=$
3879.5 ± 0.7 MeV

1. X(3872) lays above $D\bar{D}$ threshold, but does not decay into $D\bar{D}$
2. X(3872) seems to be a very narrow state
3. X(3872) lies right at, or just below, $D^0\bar{D}^{0*}$ threshold
4. X(3872) lies below $D^+D^{*-}$ threshold




It must be a molecular state comprised of $\overline{D^0}D^{0*}$ !!! Finally!

# Is there anything special about X(3872)?

X(3872) possesses several curious features:

$m(D^0+D^{0*})=$ $3871.5 \pm 0.5$ MeV
$m(D^{+}+D^{-*})=$ $3879.5 \pm 0.7$ MeV

1. X(3872) lays above $D\bar{D}$ threshold, but does not decay into $D\bar{D}$

   Wrong quantum numbers!

2. X(3872) seems to be a very narrow state

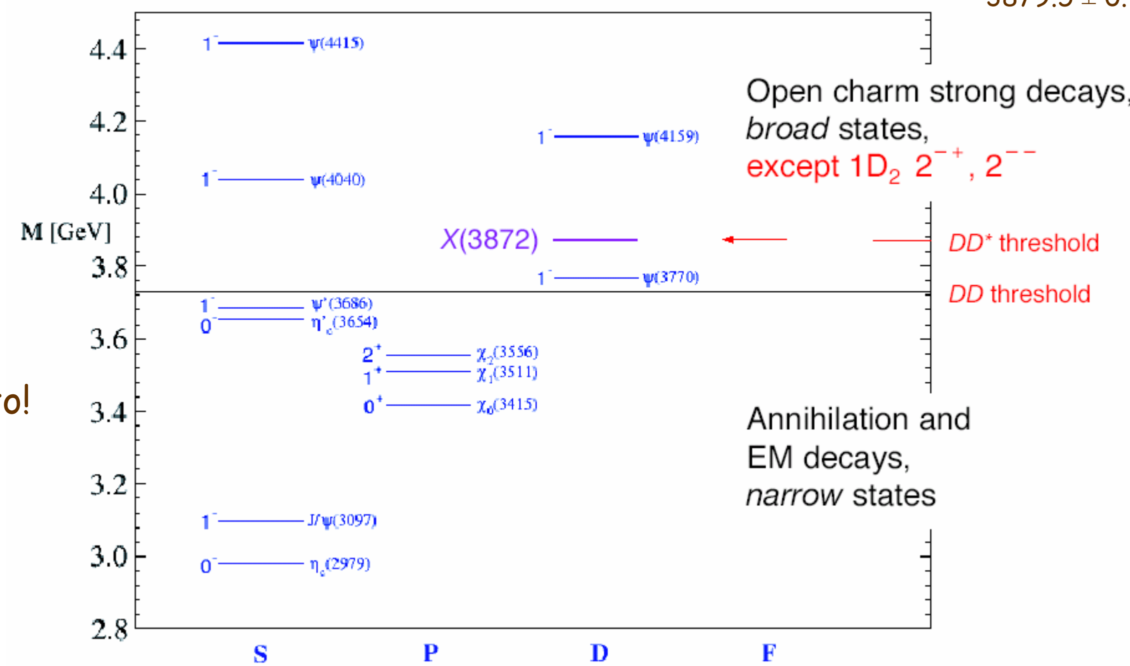
   It does NOT decay to $D\bar{D}$. There is nothing to decay to!

3. X(3872) lies right at, or just below, $D^0D^{0*}$ threshold

   Coincidence?

4. X(3872) lies below $D^+D^{*-}$ threshold

   Coincidence?

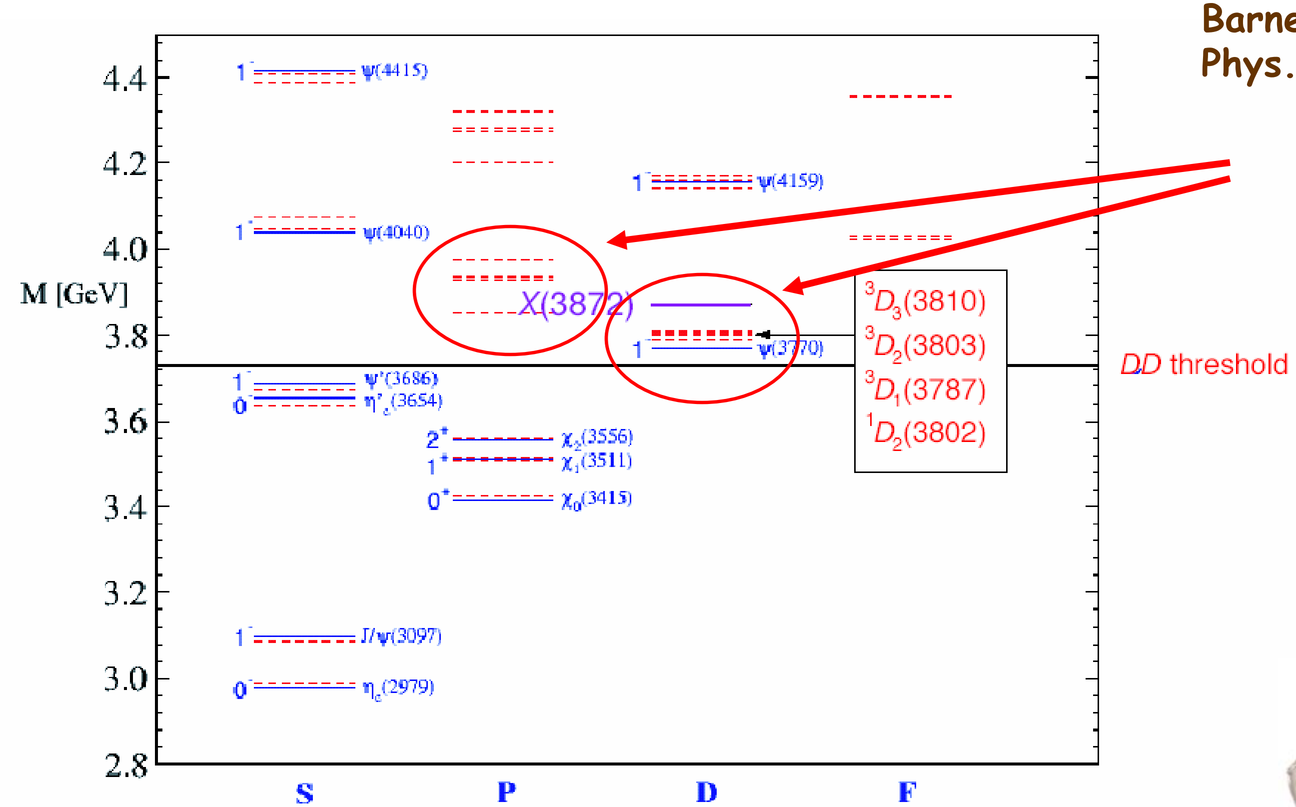



It must be an old good charmonium state… nothing exciting…

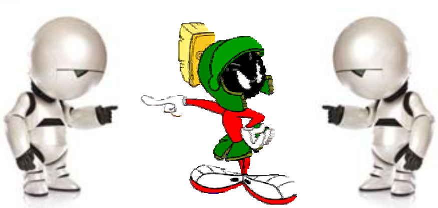
# Is there a charmonium state at 3872 MeV?

No solution for soft QCD -- must use some quark model…
Use potential model: Coulomb plus scalar confining potential

Barnes, Godfrey, Swanson
Phys.Rev.D72:054026,2005




1. Plenty of predicted states around required 3872 MeV
2. How good (robust) are those predictions anyway? Those are only quark models…
3. Lattice???



# What kind of molecule could it be?

$M_x$ = (3871.9 ± 0.5) MeV is right at the $D^0\overline{D}^{*0}$ threshold (3871.3 ± 1) MeV

$\rightarrow$ Speculation: X might be a molecule - like $D^0\overline{D}^{*0}$ bound state

**Tornquist:** $J^{PC} = 0^{-+}, 1^{++}$ $\quad$ C = +1

$\pi^+\pi^- J/\psi$ via $\rho^0 J/\psi$ intermediate state $\rightarrow m_{\pi+\pi-}$ concentrated at high masses

**Swanson:** dynamical quark model for X as a $D^0\overline{D}^{*0}$ hadronic resonance

$J^{PC} = 1^{++}$ is favored

$D^0\overline{D}^{*0}$ + admixture of $\omega J/\psi$ + small $\rho J/\psi$

These are either quark model or pion exchange models that can be tuned to obtain $M_{molecule}(X)$~3872 MeV

Need a model-independent analysis !!!

# Theoretical framework

Idea: do NOT try to predict molecular state at 3872 MeV.
Instead, ASSUME that X(3872) is a molecule and work out <u>model-independent consequences</u>.

Strategy:

1. Quantum Mechanics: poles of scattering amplitude = bound states

   Compute $\overline{D}^0 D^{0*} \to \overline{D}^0 D^{0*}$ scattering amplitude

2. Employ chiral and heavy-quark symmetries to write an effective Lagrangian

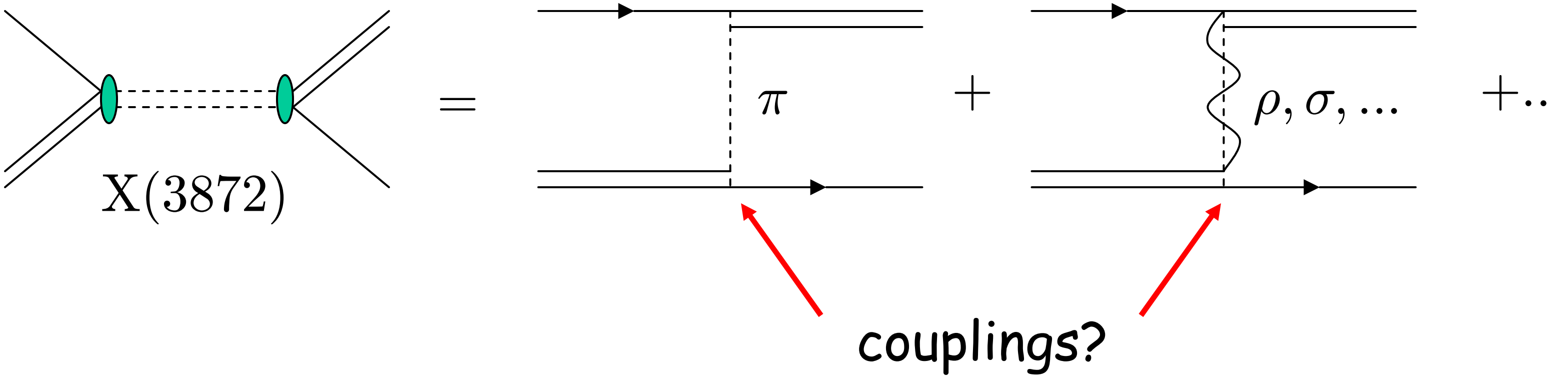
   Symmetries restrict the form of interactions

3. Compute bound state energy and use heavy-quark symmetry to relate charm and beauty systems

   If X(3872) is a molecular state: predict the presence/absence of a molecular state in $\overline{B}B^*$ channel!

# Theoretical framework

Physically, binding can be done by pions or other, heavier, particles exchanged between $D^{0*}$ and $\overline{D}^0$.




Problems:
1. Except for $D^*D\pi$, all other couplings are unknown…
2. Meson spectrum is not known well for $m_{meson}$ ~ 1 GeV…

Observation: if X(3872) is a molecule, its binding energy is

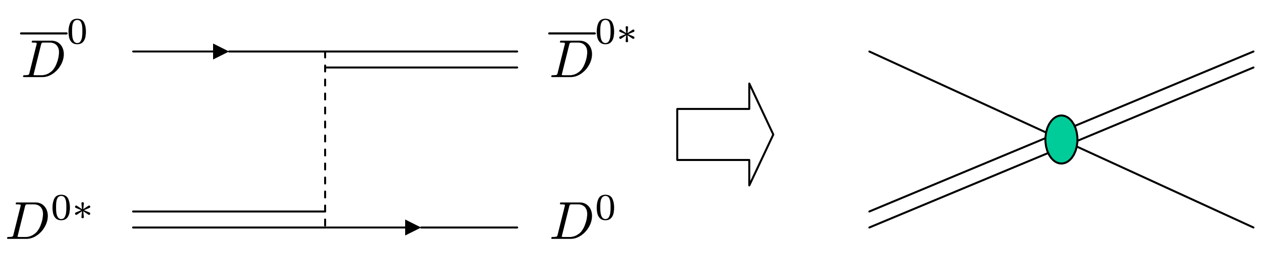
$$E_b = (m_{D^0} + m_{D^{0*}}) - M_X = -0.6 \pm 1.1 \text{ MeV}$$

# Theoretical setup

If X(3872) is a molecule, its binding energy is VERY SMALL!

$$|E_b| = -0.6 \pm 1.1 \text{ MeV} \ll m_\pi \ll m_\rho, \ldots$$

This means that heavy mesons interact via contact, point-like interactions!




Compute the transition amplitude: $T_{++} = \langle X(3872)|T|X(3872)\rangle$

Build the state: $|X_\pm\rangle = \frac{1}{\sqrt{2}}\left[\left|D^*\overline{D}\right\rangle \pm \left|D\overline{D}^*\right\rangle\right]$

# Chiral Lagrangian. Two-body case.

Two-body chiral Lagrangian for heavy meson interactions is known

$$\mathcal{L}_2 = -i\mathrm{Tr}\left[\overline{H}^{(Q)} v\cdot D H^{(Q)}\right] - \frac{1}{2m_P}\mathrm{Tr}\left[\overline{H}^{(Q)} D^2 H^{(Q)}\right]$$
$$+ \frac{\lambda_2}{m_P}\mathrm{Tr}\left[\overline{H}^{(Q)}\sigma^{\mu\nu}H^{(Q)}\sigma_{\mu\nu}\right] + \frac{ig}{2}\mathrm{Tr}\overline{H}^{(Q)}H^{(Q)}\gamma_\mu\gamma_5\left[\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right] + \dots$$

where the superfields $H^{(Q)}$ are

$$H_a^{(Q)} = \frac{1+\not{v}}{2}\left[P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5\right], \qquad \overline{H}^{(Q)a} = \gamma^0 H_a^{(Q)\dagger}\gamma^0$$

They transform under chiral and heavy-quark spin symmetries as

$$H_a^{(Q)} \to S\left(H^{(Q)}U^\dagger\right)_a, \qquad \overline{H}^{(Q)a} \to \left(U\overline{H}^{(Q)}\right)^a S^{-1}$$

# Chiral Lagrangian. Four-body case.

Four-body chiral Lagrangian for heavy meson interactions can be written by requiring the invariance under chiral and HQ symmetries

$$-\mathcal{L}_4 = \frac{C_1}{4}\mathrm{Tr}\left[\overline{H}^{(Q)} H^{(Q)}\gamma_\mu\right]\mathrm{Tr}\left[H^{(\overline{Q})}\overline{H}^{(\overline{Q})}\gamma^\mu\right]$$ ← "+" parity "exchange"

$$+\frac{C_2}{4}\mathrm{Tr}\left[\overline{H}^{(Q)} H^{(Q)}\gamma_\mu\gamma_5\right]\mathrm{Tr}\left[H^{(\overline{Q})}\overline{H}r^{(\overline{Q})}\gamma^\mu\gamma_5\right]$$ ← "-" parity "exchange"

Note: (1) other Dirac structures give identical contributions
(2) describes interactions of all DD, D*D, and D*D* states

In the case of DD* molecule:

$$\mathcal{L}_{4,PP^*} = -C_1 P^{(Q)\dagger} P^{(Q)} P_\mu^{*(\overline{Q})\dagger} P^{*(\overline{Q})\mu} - C_1 P_\mu^{*(Q)\dagger} P^{*(Q)\mu} P^{(\overline{Q})\dagger} P^{(\overline{Q})}$$
$$+C_2 P^{(Q)\dagger} P_\mu^{*(Q)} P^{*(\overline{Q})\dagger\mu} P^{(\overline{Q})} + C_2 P_\mu^{*(Q)\dagger} P^{(Q)} P^{(\overline{Q})\dagger} P^{*(\overline{Q})\mu} + \ldots$$

Note: two couplings (cf. Braaten and Kusinoki)

# A one-page calculation…

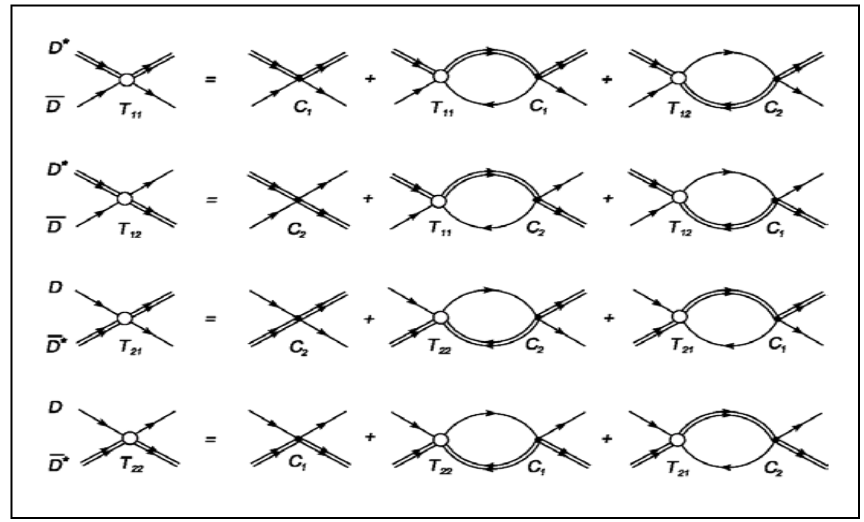
Four scattering amplitudes must be computed and related to X(3872)

$$T_{11} = \langle D^* \overline{D} | T | D^* \overline{D} \rangle,$$

$$T_{12} = \langle D^* \overline{D} | T | D \overline{D}^* \rangle,$$

$$T_{21} = \langle D \overline{D}^* | T | D^* \overline{D} \rangle,$$

$$T_{22} = \langle D \overline{D}^* | T | D \overline{D}^* \rangle$$



These are coupled Lippmann-Schwinger equations

$$\left\{ \begin{array}{l} iT_{11} = -iC_1 + \int \frac{d^4q}{(2\pi)^4} T_{11} G_{PP^*} C_1 - \int \frac{d^4q}{(2\pi)^4} T_{12} G_{PP^*} C_2, \\ iT_{12} = \quad iC_2 - \int \frac{d^4q}{(2\pi)^4} T_{11} G_{PP^*} C_2 + \int \frac{d^4q}{(2\pi)^4} T_{12} G_{PP^*} C_1, \\ iT_{21} = \quad iC_2 + \int \frac{d^4q}{(2\pi)^4} T_{21} G_{PP^*} C_1 - \int \frac{d^4q}{(2\pi)^4} T_{22} G_{PP^*} C_2, \\ iT_{22} = -iC_1 - \int \frac{d^4q}{(2\pi)^4} T_{21} G_{PP^*} C_2 + \int \frac{d^4q}{(2\pi)^4} T_{22} G_{PP^*} C_1 \end{array} \right.$$

# Results I

Four scattering amplitudes are related to X(3872) as

$$T_{++} = \frac{1}{2}\left(T_{11}+T_{12}+T_{21}+T_{22}\right)$$
$$= \frac{\lambda_R}{1+(i/8\pi)\lambda_R\,\mu_{DD^*}|\vec{p}|\sqrt{1-2\mu_{DD^*}\Delta/\vec{p}^{\,2}}}$$

$\lambda_R = (C_2 - C_1)_R$

renormalized coupling

reduced mass

$\Delta = m_{D^*} - m_D$

Extracting the pole can obtain binding energy

$$\boxed{E_b = \frac{32\pi^2}{\lambda_R^2 \mu_{DD^*}^3}}$$

which implies $\lambda_R \simeq 8.4\times 10^{-4}\ \mathrm{MeV}^{-2}$

How do we relate $E_b$ in charm and beauty???

# Results II

Argument:

1. System of two heavy particles requires nonrelativistic, not 1/M expansion:

   Powercount: $p^0 \sim \vec{p}^2/M$ in all propagators

2. Since action S does not scale with the heavy quark mass

$$S = \int d^4x \, \mathcal{L} \longrightarrow \mathcal{L} \sim 1/M$$

$$d^4x \sim M$$

3. From the kinetic term: $\mathcal{L}_2 = -\frac{1}{2m_P} \mathrm{Tr}\left[\overline{H}^{(Q)} D^2 H^{(Q)}\right]$

$$H^{(Q)} \sim M^0$$

4. From $\mathcal{L}_4 = -\frac{C_1}{4} \mathrm{Tr}\left[\overline{H}^{(Q)} H^{(Q)} \gamma_\mu\right] \mathrm{Tr}\left[H^{(\overline{Q})} \overline{H}^{(\overline{Q})} \gamma^\mu\right]$

$$\longrightarrow \boxed{C_i \sim 1/M}$$

# Results II

Since we know heavy-quark scaling of $C_i$…

$$\boxed{C_i \sim 1/M}$$

…can relate couplings for charm and beauty…

$$\lambda_R^B \simeq \lambda_R^D \frac{\mu_{DD*}}{\mu_{BB*}}$$

…so the binding energy and mass of the B-state are

$$E_b = 0.18\, MeV, \quad \boxed{M_{X_b} = (m_B + m_{B*}) - E_b = 10604\, MeV}$$

M. AlFiky, F. Gabbiani, A.A.P.
hep-ph/0506141

# Conclusions

- We proposed a model-independent description of X(3872)
- Based on heavy-quark symmetry, we predicted a new state $X_b(10604)$
- Any troubles for molecular interpretation of X(3872)?
  Isospin in B-decays… Prompt X(3872) production…